How Computer Work
Lecture 10

Introduction to the Physics of Communication

The Digital Abstraction Part 1:
The Static Discipline

Tx

Rx

Noise

\[ V_{di} \quad V_{dh} \]

\[ V_{oi} \quad V_{oh} \]
What is Information?

Information Resolves ______________

How do we measure information?

Error-Free data resolving 1 of 2 equally likely possibilities = ___________ of information.

1 bit
How much information now?

3 independent coins yield 3 bits of information

# of possibilities = 8

How about N coins?

N independent coins yield

# bits = _____________

# of possibilities = ___________
What about Crooked Coins?

\[ P_{\text{tail}} = 0.25 \quad P_{\text{head}} = 0.75 \]

\[ \text{# Bits} = - \sum p_i \log_2 p_i \]

(about .81 bits for this example)

How Much Information?

\[ \ldots 00000000000000000000000000000 \ldots \]

None (on average)
How Much Information Now?

...0101010 1010101010101...

...0101010 1010101010101...

Predictor

None (on average)

How About English?

- 6.JQ4 ij a vondurfhl co8rse wibh sjart sthdenjs.
- If every English letter had maximum uncertainty, average information / letter would be $\frac{\log_2(26)}{}$.
- Actually, English has only 2 bits of information per letter if last 8 characters are used as a predictor.
- English actually has 1 bit / character if even more info is used for prediction.
Data Compression

Lot’s O’ Redundant Bits

Encoder

Fewer Redundant Bits

Decoder

Lot’s O’ Redundant Bits

An Interesting Consequence:

- A Data Stream containing the most possible information possible (i.e. the least redundancy) has the statistics of ___________________ !!!!!

Random Noise

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Digital Error Correction

Original Message

Encoder

Original Message + Redundant Bits

Corrector

Original Message

How do we encode digital information in an analog world?

Once upon a time, there were these aliens interested in bringing back to their planet the entire library of congress ...
The Effect of “Analog” Noise

Max. Channel Capacity for Uniform, Bounded Amplitude Noise

Max # Error-Free Symbols = \( \frac{P}{N} \)

Max # Bits / Symbol = \( \log_2\left(\frac{P}{N}\right) \)
Max. Channel Capacity for Uniform, Bounded Amplitude Noise (cont)

\[ C = \frac{W \log_2(P/N)}{P} \]

P = Range of Transmitter’s Signal Space
N = Peak-Peak Width of Noise
W = Bandwidth in # Symbols / Sec
C = Channel Capacity = Max. # of Error-Free Bits/Sec

Note: This formula is slightly different for Gaussian noise.

Further Reading on Information Theory

The Mathematical Theory of Communication,

Coding and Information Theory, Richard Hamming,
The mythical equipotential wire

But every wire has parasitics:

\[ V = L \frac{dI}{dt} \]

\[ I = C \frac{dV}{dt} \]
Why do wires act like transmission lines?

Signals take time to propagate

Propagating Signals must have energy

Inductance and Capacitance Stores Energy

Without termination, energy reaching the end of a transmission line has nowhere to go - so it echoes

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Fundamental Equations of Lossless Transmission Lines

\[
V = V(x, t)
\]

\[
\frac{\partial V}{\partial x}
\]

\[
l = l(x, t)
\]

\[
\frac{\partial l}{\partial x}
\]

\[
\frac{dL}{dx} = \frac{dC}{dx} = \frac{\partial V}{\partial x} = \frac{\partial l}{\partial t}
\]

---
Transmission Line Math

Let's try a sinusoidal solution for \( V \) and \( I \):

\[
V = V_0 e^{j(\omega t + \alpha x)} = V_0 e^{j\omega t} e^{j\alpha x},
\]

\[
I = I_0 e^{j(\omega t + \alpha x)} = I_0 e^{j\omega t} e^{j\alpha x}.
\]

\[
\frac{\partial V}{\partial x} = j \Omega_x V_0 = l \Omega_x I_0
\]

\[
\frac{\partial I}{\partial x} = \frac{c}{l} \frac{\partial V}{\partial t} = j \Omega_x I_0 = c j \Omega_x V_0
\]

Transmission Line Algebra

\[
j \Omega_x V_0 = l \Omega_x I_0, \quad \Omega_x V_0 = l \Omega_x I_0
\]

\[
j \Omega_x I_0 = c \Omega_x V_0, \quad \Omega_x I_0 = c \Omega_x V_0
\]

\[
\Omega_x = \frac{1}{\sqrt{l/c}}, \quad \frac{V_0}{I_0} = \sqrt{\frac{l}{c}}
\]

Propagation Velocity \( \Omega_x \)

Characteristic Impedance \( \frac{V_0}{I_0} \)
Parallel Termination

Series Termination
Series or Parallel?

- **Series:**
  - No Static Power Dissipation
  - Only One Output Point
  - Slower Slew Rate if Output is Capacitively Loaded

- **Parallel:**
  - Static Power Dissipation
  - Many Output Points
  - Faster Slew Rate if Output is Capacitively Loaded

- **Fancier Parallel Methods:**
  - AC Coupled - Parallel w/o static dissipation
  - Diode Termination - "Automatic" impedance matching

When is a wire a transmission line?

\[ t_{fl} = \frac{l}{v} \]

Rule of Thumb:

\[ t_r < 2.5 t_{fl} \quad t_r > 5 t_{fl} \]

Transmission Line    Equipotential Line
Making Transmission Lines On Circuit Boards

- Copper Trace
- Insulating Dielectric
- Voltage Plane
- $Z_0 \propto \frac{h}{(w \sqrt{\varepsilon_r})}$
- $l \propto \frac{h}{w}$

Actual Formulas

- Microstrip:
  \[ Z_0 = \frac{50}{\sqrt{\varepsilon_r} \left( \frac{b + 2\tan \theta}{b} \right)^{0.5}} \text{ ohms} \]
  \[ \tan \theta = 1.017 \tan \frac{\theta_{	ext{dep}}}{2} \text{ rad} \]

- Stripline:
  \[ Z_0 = \frac{50}{\sqrt{\varepsilon_r} \left( \frac{b + 2\tan \theta}{b} \right)^{0.5}} \text{ ohms} \]
  \[ \tan \theta = 1.017 \tan \frac{\theta_{	ext{dep}}}{2} \text{ rad} \]
A Typical Circuit Board

1 Ounce Copper

\[ w = 0.15 \text{ cm} \]
\[ t = 0.0038 \text{ cm} \]
\[ h = 0.038 \text{ cm} \]

\[ c = 1.9 \text{ pF/cm} \]
\[ l = 2.75 \text{ nH/cm} \]

\[ Z_o = 38 \text{ } \Omega \]
\[ \nu = 1.4 \times 10^{10} \text{ cm/sec} = 14 \text{ cm/ns} \]