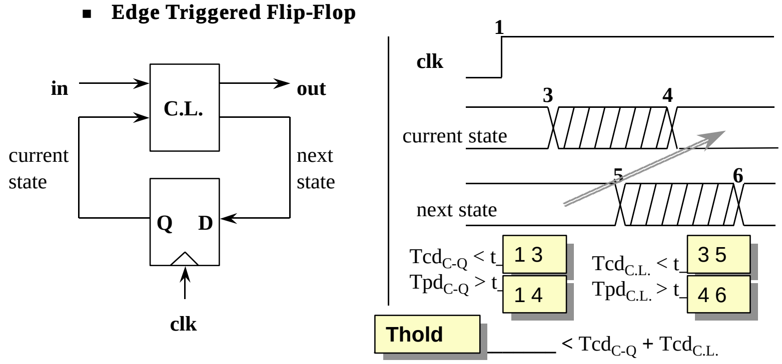
How Computers Work

Lecture 8

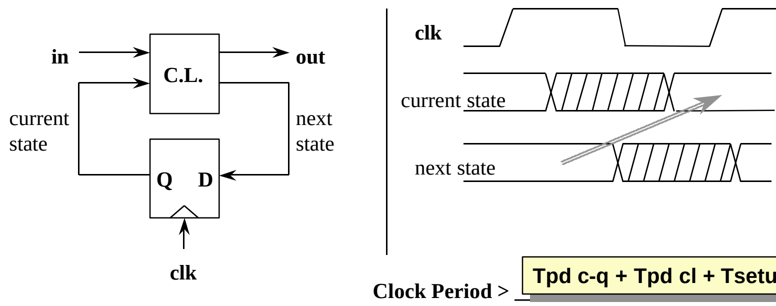
Asynchronous State Machines and Metastability

Review: Edge-Triggered Timing Constraints

- **Edge Triggered Flip-Flop**

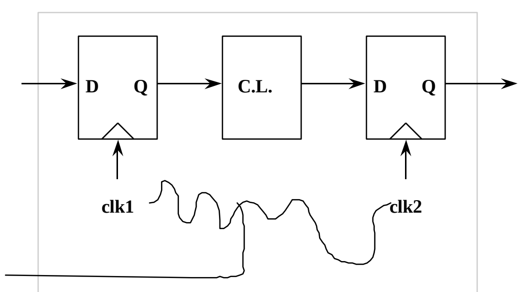


Maximum Frequency



Clock Period > Tpd c-q + Tpd cl + Tsetup

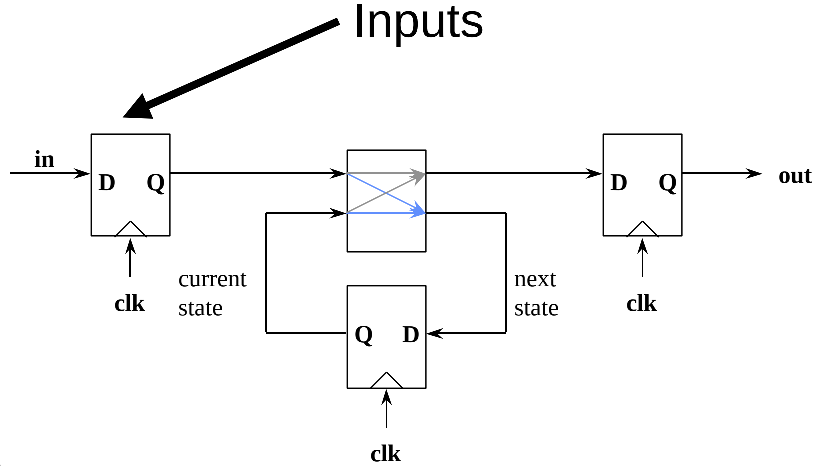
Skew



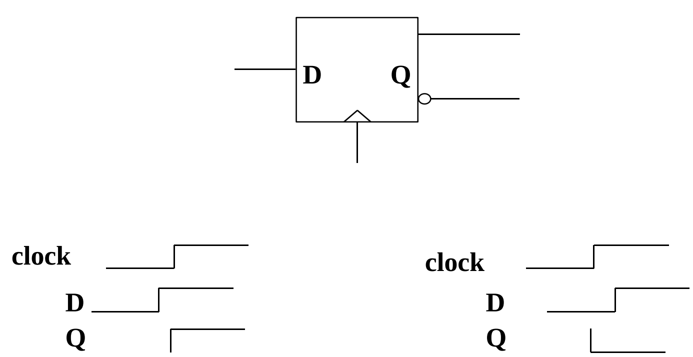
Thold + Tskew < Tcd c-q + tcd cl

Clock Period > Tpd c-q + Tpd cl + Tsetup + Tskew

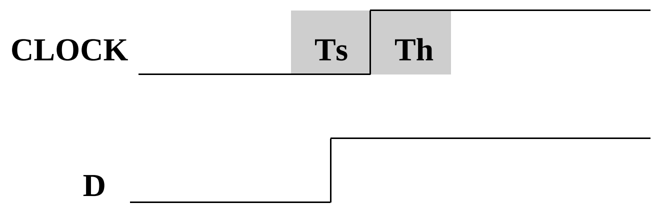
What we will cover today: The Trouble with Asynchronous Inputs



A Synchronizer as Arbiter



What Happens Now?



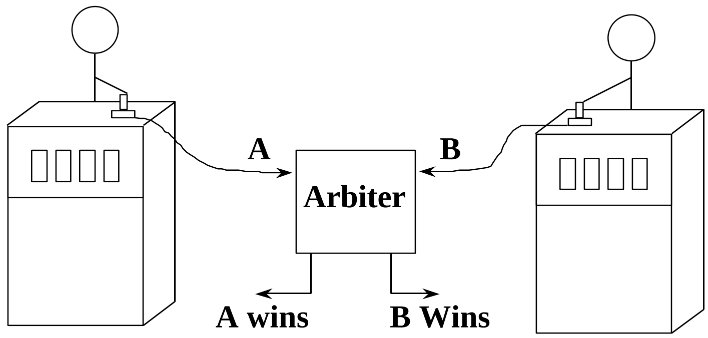
Flip-Flop Specification Says: Nothing

Buridan's Ass

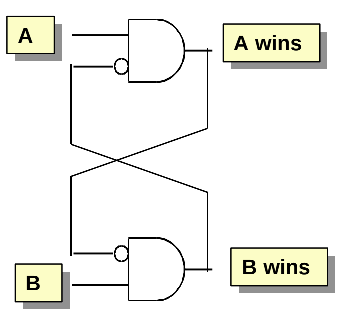
Buridan, Jean

Buridan, Jean (1300-58), French Scholastic philosopher, who held a theory of determinism, contending that the will must choose the greater good. Born in Bethune, he was educated at the University of Paris, where he studied with the English Scholastic philosopher William of Ockham. After his studies were completed, he was appointed professor of philosophy, and later rector, at the same university. Buridan is traditionally but probably incorrectly associated with a philosophical dilemma of moral choice called "Buridan's ass." In the problem an ass starves to death between two alluring bundles of hay because it does not have the will to decide which one to eat.

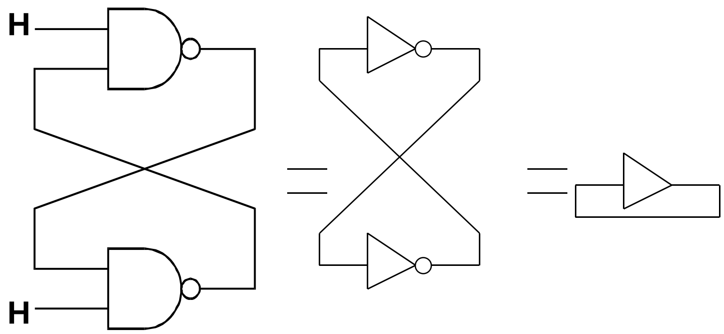
The Game Show Arbiter



A Simpler Arbiter



With Both Inputs High



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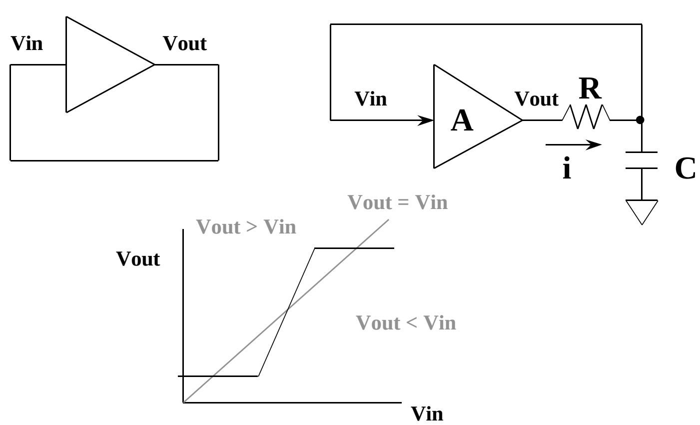
Static Metastability



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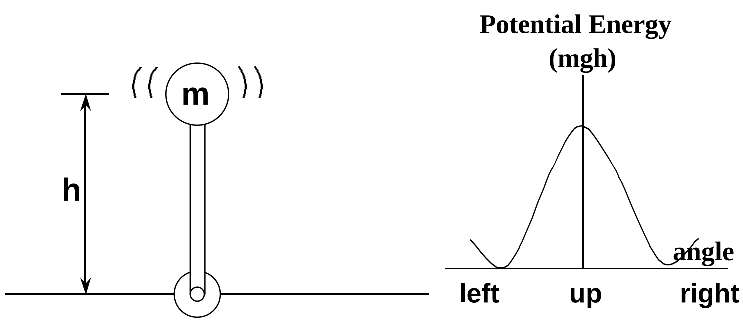
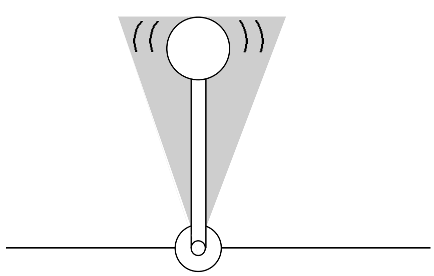
Static Metastability: Inverted Pendulum Analogy

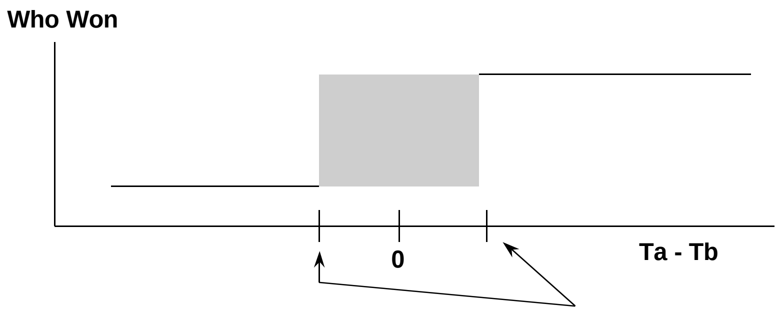


What does Heisenberg Say ?

$$\Delta x \quad \Delta p \quad > \quad h$$



What if we relax accuracy?



Arbiter output perfectly correct if | Ta - Tb | > Tmargin
if | Ta - Tb | <= Tmargin, output can be RANDOM

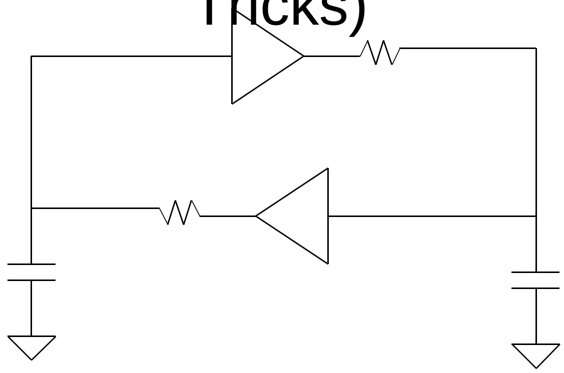
The Remarkable Fact

The perfect arbiter:

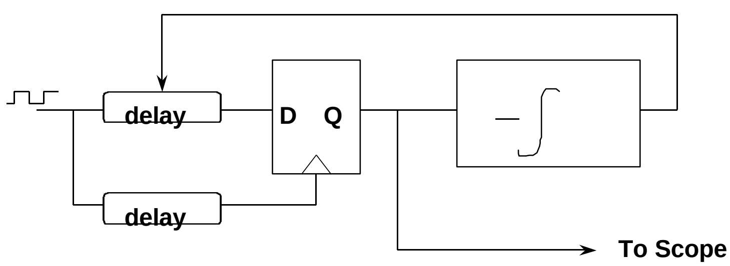
T_{margin} and T_{pd} finite and constant
Correct Ouput if $| T_a - T_b | > T_{margin}$
Any Output if $| T_a - T_b | <= T_{margin}$
Answer valid and stable after T_{pd}

IT IS IMPOSSIBLE TO BUILD ! ! ! !

Dynamic Metastability (More Stupid Classroom Tricks)

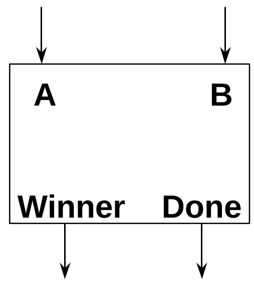


Lecture Demonstration Circuit



Is It Possible to Build This?

The Asynchronous Arbiter:



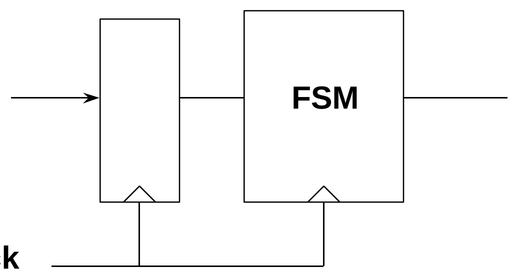
Answer:

Yes

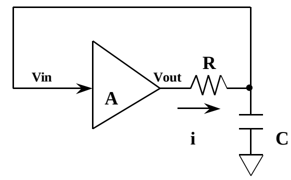
Practical Metastability

Async. Inputs

Clock

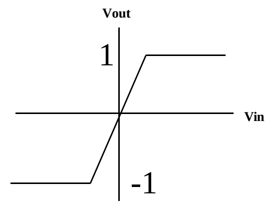


A Simple Model of Static Metstability



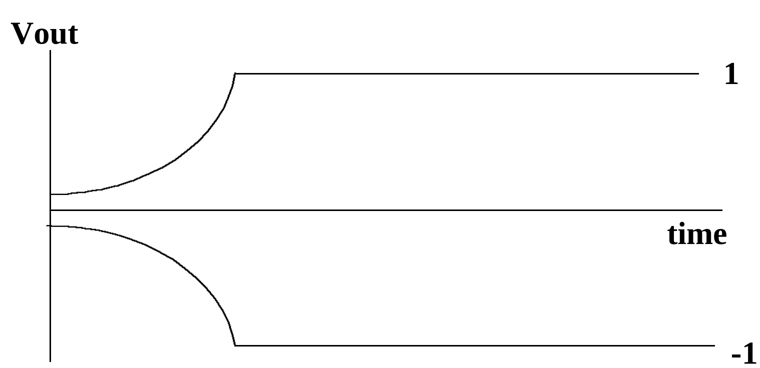
Assume Vout = A Vin (i.e. not saturated)

i = $\boxed{(A - 1)\ Vin / R}$



Vin(t) = $\boxed{E^{t(A-1)/(RC)}}$

Evolution of Vout



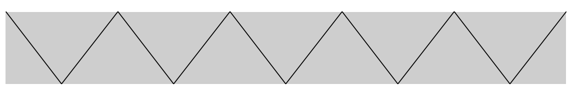
How Small must Vin(0) be to make time to saturation take longer than time t?

$$V_{in}(t) = V_{in}(0)e^{t/t} = \frac{1}{A}$$

$$V_{in}(0) =$$ 1/(A e $^{t/tau}$) = e $^{-t/tau}$ / A

If Vin(0) is uniformly distributed:

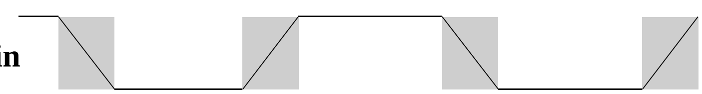
Vin



p $_{metastable}$ (t) = 1/(A e $^{t/tau}$) = e $^{-t/tau}$ / A

But if Vin is in linear rise/fall a certain fraction (p $_{transition}$) of the time:

Vin



p $_{metastable}$ (t) = p $_{transition}$ /(A e $^{t/tau}$) = (p $_{transition}$ / A) e $^{-t/tau}$

How much time do we need to achieve a certain $p_{metastable}$?

$$P_{metstable}(t) = \frac{P_{transition}}{A} e^{-t/t}$$

$$e^{t/t} = \frac{P_{transition}}{A \times P_{metstable}(t)}$$

$$t = t \ \ln\left(\frac{P_{transition}}{A \times P_{metstable}(t)}\right)$$

Example: How Long for 1 failure / year?

100 MHz Clock (t = 10 ns)

$p_{transition}$ = 0.1

A = 10

τ = 1 ns

$P_{metstable}$ = 1 / (100MHz . 1 year)

= 1 / (10^8 x π x 10^{7}) = π x 10^{-16}

$$P_{metstable} = \frac{P_{transition}}{A} e^{-t/t}$$

$$t = t \ \ln\left(\frac{P_{transition}}{A \times P_{metstable}}\right)$$

t = 10^{-9} ln(0.1 / (10 x π x 10^{-16})) = about 31 ns

How about 10 years instead of 1?

t = (10^-9)ln(pi x 10^(-15)) = 33ns

How often will failures occur if we wait 100 ns?

$$P_{metstable}(t) = \frac{P_{transition}}{A} e^{-t/t}$$

$$= \frac{0.1}{10} e^{-100} = 10^{-2} \times 10^{\frac{-100}{\ln(10)}}$$

$$\approx 10^{-45.4}$$

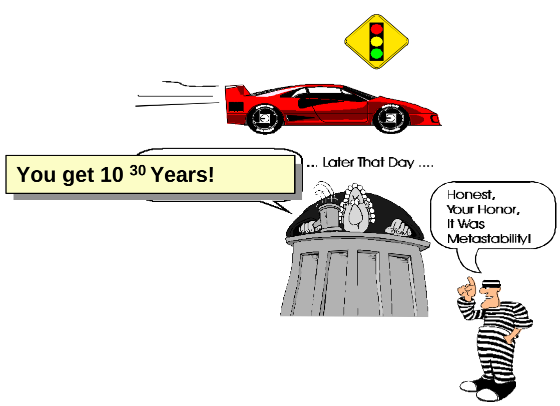
At 100 MHz, this is about 1 failure every 10^{30} **YEARS!**

Age of Hominids: 10 ^{7}

Age of Earth: 10 ^{9}

Age of Universe: 10 ^{10}

Is this a Good Excuse For Cruising a Light?



What did we Learn Today?

- If we violate setup or hold times, a flip-flop can give a random digital output.
- If we violate setup or hold times, we can't bound the propagation delay of a flip-flop.
- Metastability usually causes strange outputs, but flip-flops are sold that have valid, stable, outputs while internal nodes are metastable. They can still change their minds when coming out of metastability.
- In practice, we can choose a propagation time that will have a forever stable output "most" of the time.
- If we wait long enough (typ. 10-100 ns) "most of the time" is almost all of the time.
- We can easily detect when settling happens, but we can't say how long it will take.